

The South African Geoid 2010: SAGEOID10

by Garth Chandler, CD:NGI, and Dr. Charles Merry, consultant on GPS and Geodetic Surveys

A precise geoid model is a prerequisite for converting GPS-determined heights to heights referred to in the national height datum. This article describes the computation of a hybrid geoid model for South Africa from a combination of gravity data, harmonic coefficients and GPS/levelling data.

GPS (and other GNSS systems) provide fully three-dimensional positions – latitude, longitude and height. The reference surface used is the WGS84 ellipsoid, and heights are heights above this surface – ellipsoidal heights. However, the type of height that is needed is a height above the geoid, which corresponds closely to mean sea level (MSL). The difference between these two types of heights varies spatially (it can reach 120 m) and is equivalent to the separation between the geoid and the ellipsoid – the geoidal height.

If a precise model can be created of this separation then it becomes possible to convert GPS-determined heights to heights above the levelling datum. This is where the difficulty arises – it is extremely difficult to determine such a model, or at least to determine an accurate model. A previous paper [1] in this journal described some of the available models. Unfortunately none of them are sufficiently accurate or detailed.

As part of its mandate to provide spatial information to support land development projects in South Africa, the Chief Directorate: National Geo-spatial Information (NGI) commissioned a study to develop an accurate hybrid geoid model for South Africa. This model had to be capable of converting ellipsoidal heights determined using NGI's TrigNet to orthometric heights on the South African Land Levelling Datum, to an accuracy of 10 cm.

The coordinates of the TrigNet base stations are defined on the WGS84 reference ellipsoid in the ITRF2005(2010.02) reference frame. The "2010.02" means that the coordinates are updated from the

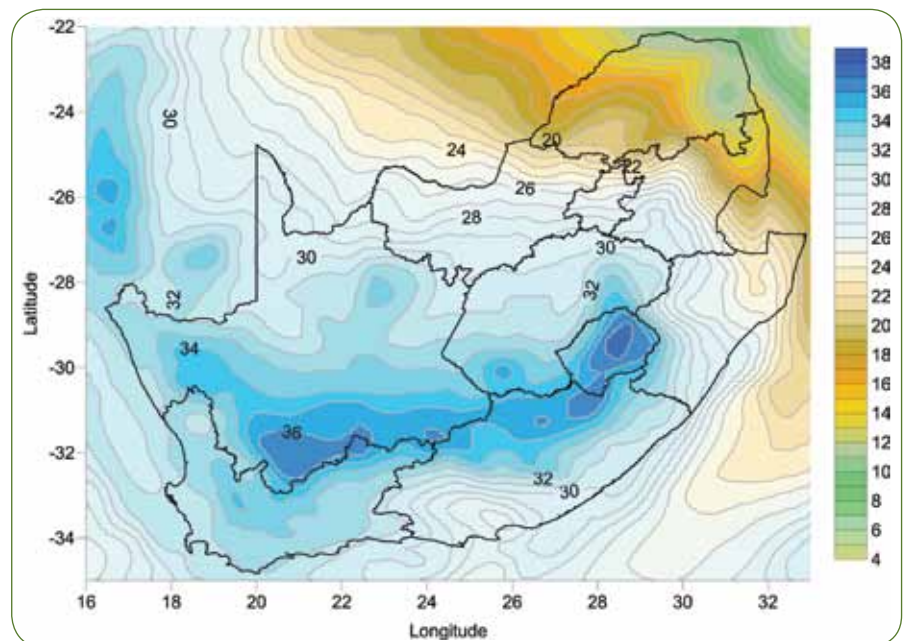


Fig. 1: Gravimetric geoid model for South Africa (metres).

beginning of the year 2005 to the date 2010.02, allowing for crustal motion. Use of the TrigNet system (real-time or post-processed GPS) will result in coordinates in this system. The published heights of all benchmarks, town survey marks and trigonometrical beacons are given as heights above LLD. This datum is close to, but not exactly the same as the geoid. Differences exist due to a number of causes, including:

- Errors in the tide gauge measurements used to define the datum.
- Systematic rise in MSL since the tide gauge measurements made about a century ago.
- Errors due to the approximate modelling of gravity effects on the precise level network.

- Errors due to the approximate adjustment techniques used for the precise levelling.

All the above effects mean that even were a perfect geoid model to be created it would not provide an exact means of transforming heights from one datum to another. Any geoid model will, in fact, not be perfect and will be subject to both random and systematic errors due to inadequacies in the data coverage and imperfect modelling. In the following sections we describe the determination of a gravimetric geoid model, and its subsequent conversion to a hybrid geoid model which will closely approximate such a transformation.

Gravimetric geoid

A combination approach, described in [2] has been used. Here, a spherical

harmonic expansion of the Earth's geopotential, derived largely from the GRACE satellite mission, has been used to model the long wavelength component of the geoid. To be more specific, the EGM2008 geopotential [3], truncated at degree 360 has been used. The residual (high frequency) part of the geoid has been determined from a combination of land and marine gravity anomalies and a digital elevation model. In more detail, the modelling equations are:

$$N = N_L + N_S \quad (1)$$

where the long wavelength component N_L is obtained from the EGM2008 harmonic coefficients (see Eqn. 2).

The short wavelength contribution, using both land and marine gravity anomalies, together with a terrain correction, is given by the following convolution integral:

$$N_S = \frac{R \cdot \Delta\phi \cdot \Delta\lambda}{4\pi\gamma} [S(\psi) * \Delta g_r \cos \phi] \quad (3)$$

where the reduced anomalies Δg_r are derived from the observed free-air gravity anomalies, Δg_f :

$$\Delta g_r = \Delta g_f - \Delta g_L + G_1 \quad (4)$$

Here the term Δg_L is computed from the same harmonic coefficients used in equation (2), (see Eqn. 5).

The terrain correction G_1 is determined from a convolution of heights with gravity anomalies, (see Eqn. 6).

$$G_1 = \frac{\Delta\phi \cdot \Delta\lambda}{2\pi} \left[(h \cdot \Delta g_f) * \frac{1}{\ell^3} - h \cdot (\Delta g_f * \frac{1}{\ell^3}) \right] \quad (6)$$

These equations were used to compute a gravimetric geoid on a regular $2\frac{1}{2}'$ (roughly 4,6 km) grid over South Africa (Fig. 1). The grid size was selected so as to ensure compatibility with the $2\frac{1}{2}'$ grid of the EGM2008 geoid model.

GPS/levelling

As mentioned previously, this gravimetric geoid will not provide a perfect transformation model due to errors in the levelling and in the geoid model itself. It can be improved by means of a correction surface deduced from GPS measurements made at precise levelling benchmarks. The difference between the GPS-determined ellipsoidal height, h and the levelled orthometric height, H , is the geoidal height, N :

$$N = h - H \quad (7)$$

NGI undertook a field project in 2008 to take precise GPS measurements at 79 benchmarks distributed throughout the country. Dual frequency receivers were operated for a minimum of four hours at each benchmark, and check measurements were made to neighbouring benchmarks to confirm that no local disturbance of the benchmark had taken place. These measurements, together with data collected at nearby TrigNet base stations, were used in the Bernese software to determine precise ITRF2005(2010.06) coordinates for the benchmarks. The ellipsoidal heights were differenced with the published heights to obtain geoidal heights at the 79 benchmarks (shown in red in Fig. 2). These formed the basis for the calibration of the gravimetric geoid.

In 2009, NGI occupied an additional 39 benchmarks distributed around the country. These additional data were processed in a similar fashion and provide an independent check on the quality of the calibrated (hybrid) geoid model. These points are shown in blue in Fig. 2.

Hybrid geoid model

A hybrid geoid model is one in which a geometric model such as that

created from GPS/levelling data is combined with a gravimetric model. The geometric model has high accuracy but is only defined at discrete points; the gravimetric model is continuous but is burdened by biases and tilts. The hybrid model is created by comparing the two models and using the differences to transform (calibrate) the gravimetric model. The simplest way to do this is to fit a tilted plane (or higher order surface) to the differences. However, this does not take into account the complexity of the differences, which can seldom be modelled by such a simplistic approach. Consequently the most favoured approach is to interpolate a grid of corrections from the set of differences. This correction grid would use the same grid size as the original gravimetric geoid. Such an approach has been used successfully in the USA [4], UK [5] and Australia [6].

The geoidal heights computed at the 79 benchmarks observed with GPS in 2008 were compared to the gravimetric geoid shown in Fig. 1. Summary statistics are shown in Table 1.

The mean difference of 20 cm is probably due to a combination of biases in the LLD and in the geoid model. The standard deviation is an indication of the "goodness-of-fit" of the two data sets. There is some geographical correlation of these differences, as can be seen in Fig. 3.

The surface shown in Fig. 3 is the correction surface required to convert the gravimetric geoid to a hybrid geoid consistent with the GPS/levelling data. The surface is formed by interpolating corrections onto a $2\frac{1}{2}'$ grid from the differences at the GPS/levelling benchmarks. Interpolation was performed using Kriging, in which some allowance was made for errors in the data. The effect of this is a slightly smoother surface than would be achieved were the surface to represent the data exactly. This has been done deliberately, so as to accommodate any possible local errors at the benchmarks. Nevertheless, a very close fit was obtained, with a standard deviation (σ) of 6 mm. A similar approach was used in the UK and USA, with standard deviations of 4 mm and 15 mm respectively.

Adding the correction surface to the gravimetric geoid model produces the hybrid geoid model, which takes advantage of the best features of

$$N_L = \frac{GM}{\gamma R} \sum_{n=2}^{360} \left(\frac{a}{R} \right)^n \sum_{m=0}^n (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \cdot \bar{P}_{nm}(\cos \theta) \quad (2)$$

Equation 2

$$\Delta g_L = \frac{GM}{a \cdot R} \sum_{n=2}^{n_{max}} (n-1) \left(\frac{a}{R} \right)^n \sum_{m=0}^n [(C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) \cdot P_{nm}(\cos \theta)] \quad (5)$$

Equation 5

both the gravimetric geoid and the geometric geoid represented by the GPS/levelling data

Validation

Internal validation

Although the hybrid geoid model fits well (σ of 6 mm) to the GPS/levelling data, this is no guarantee that the model will fit well at other points. As a first step in validating the model, cross validation was used. In this approach, 10% of the data (8 points) were randomly selected and removed from the data set. The remaining data (71 points) were used to generate a correction surface, which was then used to estimate corrections at the excluded 8 points. The discrepancies at these points are a measure of the ability of the correction surface to represent corrections at the excluded points. This process was repeated ten times, randomly selecting different combinations of exclusion points. The resultant root mean square (RMS) error of representation is 7 cm. This is a more realistic estimate of the accuracy of converting GPS heights to orthometric heights using the hybrid geoid.

External validation

In order to provide a totally independent assessment of the accuracy of the hybrid geoid, NGI took GPS measurements at an additional 39 benchmarks during 2009. These are shown in blue in Fig. 2. The hybrid geoid model created using the original 79 data points was then used to convert the GPS heights to orthometric heights, which could then be compared with the published heights at the 39 validation points. The results are summarised in Table 2.

The standard deviation of 7 cm confirms the results obtained in the internal validation process. The maximum discrepancy of 28 cm occurs near Port Shepstone and is a cause for some concern as its source has not been identified.

Final hybrid geoid model

Bearing in mind that the primary purpose of this project is to provide a surface that will enable users to convert GPS-derived heights to heights consistent with the current LLD, it would be foolish not to incorporate these additional 39 points into a new correction surface. Consequently, all 118 points have been used to determine such a surface. Again, Kriging was used to interpolate corrections onto a

Difference	Minimum	Maximum	Mean	Std. Dev.
GPS/levelling – gravimetric geoid	-0,66 m	0,28 m	-0,20 m	0,22 m

Table 1: Differences between GPS/levelling and gravimetric geoid model.

Difference	Minimum	Maximum	Mean	Std. Dev.
GPS/levelling – hybrid geoid model	-10 cm	28 cm	0,4 cm	7 cm

Table 2: Differences between GPS/levelling and hybrid geoid at 39 validation points.

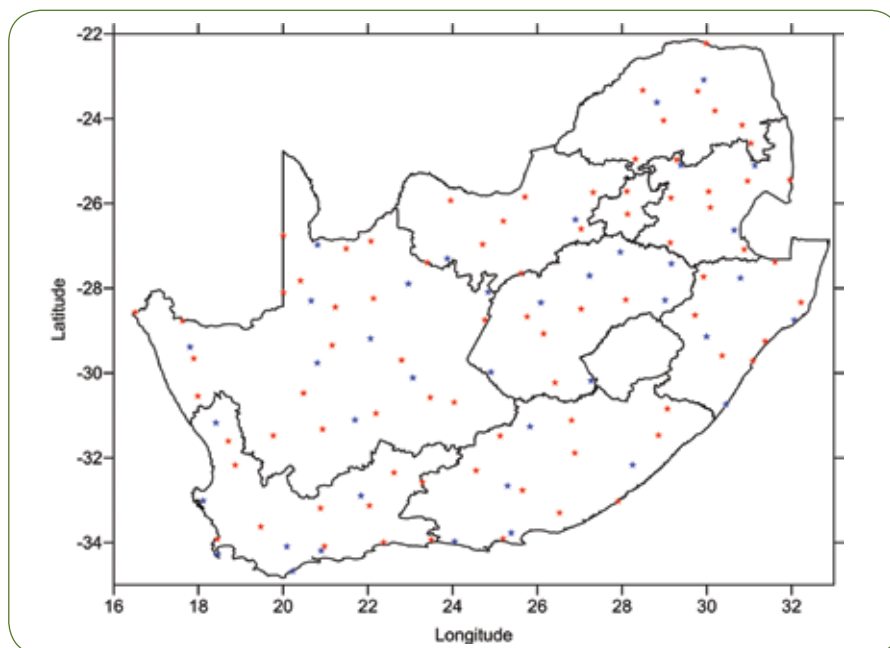


Fig. 2: GPS levelling data.

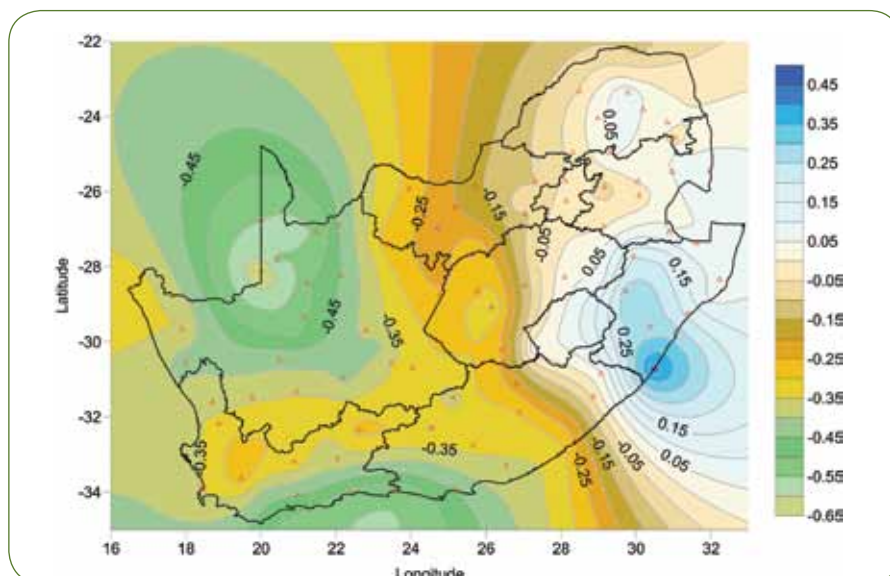


Fig. 3: Differences – GPS/levelling minus gravimetric geoid (metres).

grid. However, in this case, to allow for possible errors in the data, the surface was allowed to relax still further and the resultant standard deviation

of the fit to the data is 19 mm. Note that this just represents the fit of the correction surface to the data. A more realistic estimate for the accuracy of

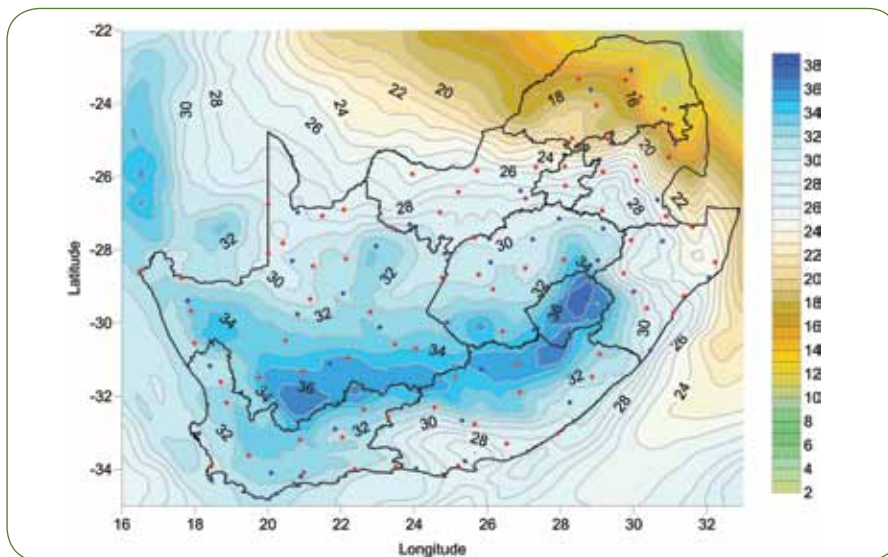


Fig. 4: Final hybrid geoid model.

transforming GPS heights to LLD is 7 cm.

The final hybrid geoid model has been labelled SAGEOID10. It will be freely available from NGI in the form of an ASCII data file, where each record consists of: latitude, longitude, geoidal height. The data are on a $2\frac{1}{2}'$ grid, in a format compatible with EGM2008 data generated using the program *gridget_2p5min*, supplied by the US National Geospatial-Intelligence Agency [7]. The data cover the region: $-35^\circ \leq \text{latitude} \leq -22^\circ$; $16^\circ \leq \text{longitude} \leq 33^\circ$.

Release notes

The geoid model is being released at this time based on the ITRF2005 (epoch 2010.02) reference frame on which TrigNet station coordinates are now based. The International Earth Rotation and Reference Systems Service (IERS) will be releasing ITRF2008 during the course of 2010 and all International GNSS Service (IGS) products including GNSS satellite orbits will be published based on ITRF2008 shortly thereafter. A consequence of the change to ITRF2008 will be that TrigNet station coordinates will be converted to the new version of ITRF and the geoid model as now being released will be aligned with the TrigNet ITRF2008 coordinates. A new version of the geoid based on ITRF2008 will be released in 2011.

The purpose of this release of the geoid based on ITRF2005 (epoch 2010.02) is to allow users to test the accuracy of the product and to comment so that the next version can be corrected or adjusted as necessary. The model has been released to GNSS receiver vendors

for implementation in their firmware and software. The format of the geoid file is identical to that of the EGM2008 and should not cause any problems with implementation on receivers. The geoid file, SAGEOID2010.dat, available as SAGEOID2010.zip and a short presentation can be downloaded from the TrigNet website on <ftp://ftp.trignet.co.za/RefData.Station%20information/SA%20Geoid> or on RefData.Station Information/SA Geoid on www.trignet.co.za

Conclusions

The aim of this project was to produce a hybrid geoid model capable of converting GPS-derived heights to the Land Levelling Datum with an accuracy of better than 10 cm. This has been achieved - the accuracy is estimated to be 7 cm.

There are a couple of provisos associated with the use of SAGEOID10:

- It is only valid within the borders of South Africa. Although the data file covers a larger region, the correction surface has been computed using GPS/levelling data within South Africa's borders only. Any attempt to extrapolate beyond these borders could lead to serious errors.
- It is only valid for the conversion of GPS-derived heights based upon TrigNet (reference frame ITRF2005(2010.06)) to the current Land Levelling Datum. If, at some future date, either the TrigNet datum or the LLD is changed, a new correction surface will be produced.

As always, this is a work in progress. Improvements in gravity data coverage or new geopotential models could lead to

a revised version of the gravimetric geoid and of the hybrid model. Additional GPS measurements at benchmarks could also lead to changes in the model.

Acknowledgments

In addition to the GPS measurements made by NGI, data were provided by a number of organisations. Chief amongst these is the Council for Geoscience, which provided gravity data for South Africa. The Geological Surveys of neighbouring countries provided additional gravity data, as did some commercial companies. The Petroleum Agency of South Africa and the Danish National Space Agency provided marine gravity data. The harmonic coefficients of the EGM2008 geopotential model were obtained from the US National Geospatial-Intelligence Agency, and the 30" SRTM digital elevation model was obtained from the US National Aeronautics and Space Administration.

References

- [1] C L Merry: "GPS and the geoid", PositionIT, July/August 2008.
- [2] C L Merry: "An updated geoid model for Africa", presented at Symposium G2, XXIV General Assembly of the IUGG, Perugia, Italy, July 2007.
- [3] N K Pavlis, S A Holmes, S C Kenyon, and J K Factor: "An Earth Gravitational Model to Degree 2160: EGM2008", presented at the 2008 General Assembly of the European Geosciences Union, Vienna, Austria, April 2008.
- [4] D R Roman, Y M Wang, J Saleh, and X Li: "National geoid height models for the United States: USGG2009 and GEOID09", www.ngs.noaa.gov/GEOID/USGG2009, accessed on 08 June 2009.
- [5] J C Iliffe, M Ziebart, P A Cross, R Forsberg, G Strykowski, and C C Tscherning: "OSGM02: A new model for converting GPS-derived heights to local height datums in Great Britain and Ireland", Survey Review, 37(290), 276:293.
- [6] W E Featherstone, D M Sproule, J M Goos, J F Kirby, M Kuhn, and S J Claessens: "Towards the new AusGeoid model", Proceedings of SSC 2005 Spatial Intelligence, Innovation and Praxis: The national biennial conference of the Spatial Sciences Institute, Melbourne, September, 2005.
- [7] National Geospatial-Intelligence Agency: "EGM2008 - WGS 84 Version", http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/egm08_wgs84.html, accessed on 09 June 2009.

Contact Garth Chandler, CD:NGI, Tel 021 685-4475, gchandler@ruraldevelopment.gov.za or Dr. Charles Merry, Tel 021 712-0042, dr.charles.merry@gmail.com ©